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Cranking in isospace

Applications to neutron-proton pairing and the nuclear symmetry energy

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Abstract. Isoscalar pairing interaction and nuclear symmetry energy are investigated by means of the iso-cranking technique. Iso-cranking represents the lowest order approximation to isospin projection after variation. Due to its internal simplicity, it offers a very intuitive understanding of the structure of the nuclear symmetry energy as well as the response of the isoscalar and isovector pairing versus isospin.

PACS. 21.60.Jz Hartree-Fock and random-phase approximations

1 Introduction

An adequate treatment of the isospin degree of freedom is crucial for our understanding of low-energy nuclear structure excitations. Hereafter, we shall present two applications of the isospin cranking model, which represents the lowest order approximation to isospin projection after variation. Because of its internal simplicity, iso-cranking offers an intuitive understanding to the underlying mechanisms of the isoscalar pairing phenomenon and allows to unveil the physical origin of the nuclear symmetry energy strength.

We will start with a brief discussion of the isovector and isoscalar pair fields different response to rotation in isospace [1]. The destructive role of the isospin degree of freedom on the isoscalar pair field and, simultaneously, the neutrality of the isovector pair field can be easily and intuitively understood via a direct analogy between rotations in isospace and real space, respectively.

In the second part we will focus on the microscopic structure of the nuclear symmetry energy (NSE) strength within the Skyrme-Hartree-Fock (SHF) method [2]. We will demonstrate that the strength of the NSE originates in part from the discreteness of the single-particle levels characterized by the mean level spacing ε which is governed essentially by the isoscalar mean-potential. We will discuss the influence of non-local effects on ε and, in turn, on the second component of the NSE, namely on the part related directly to the mean isovector potential. We will demonstrate that this part, in spite of the apparent complexity of the Skyrme mean isovector potential, can be characterized essentially by a single number, the strength of the effective isospin-isospin $v_{TT} = \frac{1}{2}\kappa\hat{\mathbf{T}}^2$ interaction, with surprisingly high accuracy.

2 Neutron-proton pairing

Isoscalar ($T = 0$) np-pairing is expected to develop in $N \approx Z$ nuclei. In these nuclei, the neutron and proton wave functions overlap most strongly and the isoscalar two-body NN interaction is on the average stronger than the isovector ($T = 1$) NN interaction [3]. However, the $T = 0$ matrix elements (m.e.) are, unlike the $T = 1$ m.e., fragmented over different spin (J) values with almost equal preference for anti-parallel $J = 1$ and parallel $J = 2j$ couplings and with sizable m.e. corresponding to intermediate J-coupling. Moreover, the ($T = 0$, J) m.e. are weaker than the dominant ($T = 1$, $J = 0$) m.e. These two facts tend to erode the potential fingerprints of the isoscalar np-paired condensate causing interpretational difficulties both on the experimental as well as theoretical side.

In deformed nuclei, however, individual features of the ($T = 0$, J) m.e. are expected to become averaged. Hence, one can assume that the basic features of nuclear pairing in deformed $N \approx Z$ nuclei can be reasonably accounted for by using a standard seniority-type $T = 1$ pairing interaction and a schematic $T = 0$ np-pairing interaction:

$$\hat{H} = \hat{h}_{sp} - G_1 \sum_{\mu=0,\pm1} P^\dagger_{1\mu}\hat{P}_{1\mu} - G_0\hat{P}^\dagger_0\hat{P}_0, \qquad (1)$$

where $P^\dagger_0 = \frac{1}{\sqrt{2}}\sum_{\alpha>0}(\hat{a}^\dagger_{\alpha n}\hat{a}^\dagger_{\bar{\alpha}p} - \hat{a}^\dagger_{\alpha p}\hat{a}^\dagger_{\bar{\alpha}n})$ denotes the $T = 0$ pair creation operator and $\hat{h}_{sp} = \sum_{\alpha,\tau=n,p} e_{\alpha\tau}\hat{a}^\dagger_{\alpha\tau}\hat{a}_{\alpha\tau}$ stands for the deformed phenomenological Woods-Saxon potential. The pure HFB (or BCS) approximation to the

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model Hamiltonian (1) essentially exclude the most interesting mixed-phases, quartetting-type solutions, see [4,5]. To obtain such a solution, the theory requires a supplementary spontaneous isospin symmetry breaking mechanism which can be introduced via the approximate particle-number symmetry projection of Lipkin-Nogami type [5].

A possible manifestation of the *np*-pairing collectivity is the so called Wigner effect, an extra binding energy in $N = Z$ nuclei. Within the mean-field approach, the Wigner energy is associated with a linear ($\sim T$) contribution to the symmetry energy:

$$E_{\rm sym} = \frac{1}{2} a_{\rm sym} T(T + \lambda). \qquad (2)$$

This term, which is indeed due to the $T = 0$ interaction as indicated by shell-model studies [6], is essentially beyond the conventional mean-field and must be introduced as a phenomenological correction. Hence, as a first approximation one can ascribe its microscopic origin to the $T = 0$ pairing which, when strong enough, can naturally account for the missing binding energy, see ref. [5]. A fit to the Wigner energy allows the determination of the unknown strength of the $T = 0$ pairing interaction G_0. The consistency of this approach can be tested later by calculating the lowest $T = 0, 1, 2$ isobaric excitations in $N = Z$ nuclei. Such an approach was undertaken in ref. [1] where an excellent agreement with the data was obtained. The key to this success was the proper treatment of both the quasi-particle (qp) excitations and the isospin degree of freedom. The latter was treated within the iso-cranking approximation to the isospin variation after projection:

$$\hat{H}^{\omega} = \hat{H} - \omega \hat{T}_x. \qquad (3)$$

The results of our calculations are presented in fig. 1. To understand the role of qp and isospin degrees of freedom let us concentrate on the case of the $T = 1$ and $T = 2$ excitations in even-even (e-e) $N = Z$ nuclei. The $T = 2$, $T_z = 0$ excitations belong to the isospin quintuplet $T = 2$, $T_z = 0, \pm 1, \pm 2$, of $J = 0$ states that include the ground states of $N - Z = \pm 4$ nuclei. Hence they are treated as 0qp HFB vacuum cranked in isospace to restore the proper value of isospin $\langle \hat{T}_x \rangle [\equiv \sqrt{T(T+1)}] = \sqrt{6}$. The $T = 1$ states, on the other hand, belong to the $T_z = 0$, ± 1 triplet of $J \neq 0$ states. Hence, their HFB treatment requires both time reversal symmetry breaking and the isospin symmetry restoration. This can be achieved by isocranking the appropriate 2qp excitation till the frequency is reached where $\langle \hat{T}_x \rangle = \sqrt{2}$.

The model predicts that the $T = 2$ states are purely isovector-paired. The isocranking frequency necessary to reach $\langle \hat{T}_x \rangle = \sqrt{6}$ is, in these cases, large enough to break the antiparallel (in isospace) coupled isoscalar pairs, causing a Meißner-type phase transition, see fig. 2. In spite of the good agreement to the data the model has a clear drawback; the isoscalar pair-gaps necessary to reproduce the data are nonphysically large. This is a direct consequence of the lack of the isovector mean-potential (in excited states of $N = Z$ nuclei) in the sp model Hamiltonian (1). Consequently, the additional binding energy due to the *np*-pairing is used partly to restore the symmetry energy strength $a_{\rm sym}$, and partly to enhance the linear term to its empirical value of $\lambda \approx 1.25$ in $N \approx Z$ nuclei [8,9,10] [1].

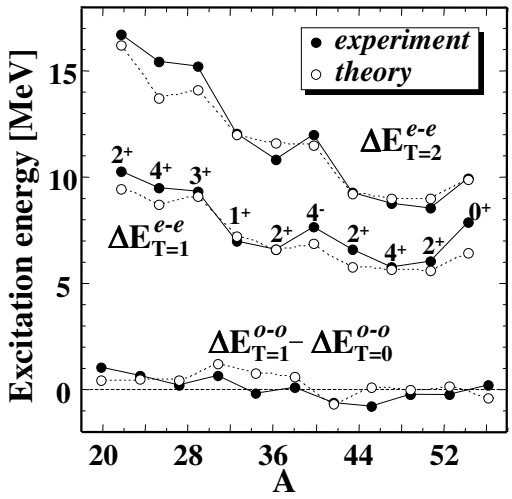

Fig. 1. The experimental (•) and calculated (○) excitation energies of the lowest $T = 2$ states and the lowest $T = 1$ states in e-e $N = Z$ nuclei, and the difference between the excitation energies of the lowest $T = 1$ and the lowest $T = 0$ states in o-o $N = Z$ nuclei, see text and refs. [1,7] for further details.

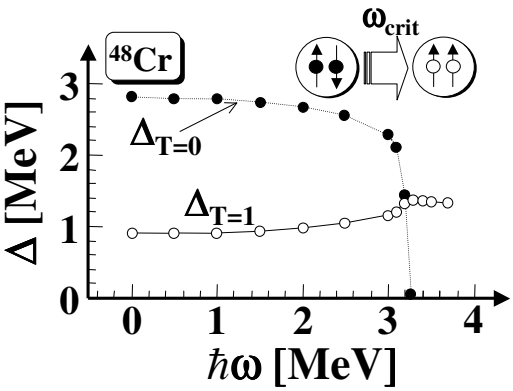

Fig. 2. Isoscalar (•) and isovector (○) pair-gap parameters versus iso-cranking frequency calculated for ^{48}Cr. The figure shows the phase transition from mixed pairing phase to purely isovector pairing phase induced by the fast iso-rotation. Note that $\Delta_{T=1}$ stays fairly constant versus ω because iso-rotation cannot break isovector pairs having parallel coupled isospins.

3 The nuclear symmetry energy

The missing isovector potential can be simulated by adding an isospin-isospin interaction, see [11], to the

[1] See also J. Jänecke, these proceedings.

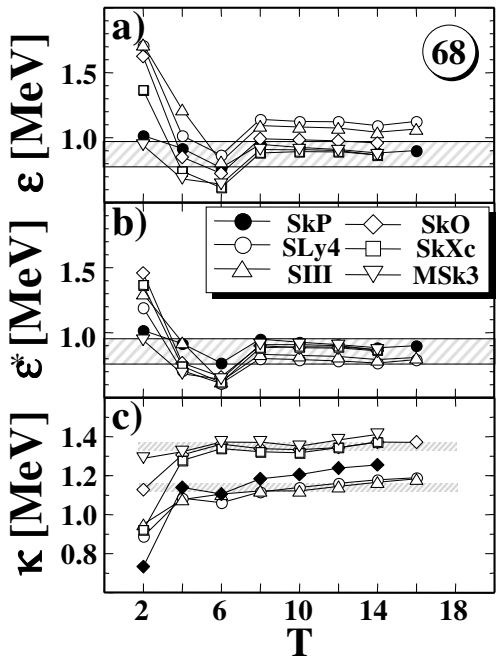

Fig. 3. Part (a) shows the mean level spacing ε calculated using the isoscalar part of the SHF potential. Part (b) shows the isoscalar effective mass scaled mean level spacing $\varepsilon^\star = \frac{m^\star}{m}\varepsilon$. Note that for larger T the value of $\varepsilon^\star$ is almost constant and lies in between the empirical limits for the mean level spacing marked by the shaded area. Part (c) shows the calculated value of $\kappa(A)$. Note that this value is almost perfectly independent of T. The calculations were done for the $A = 68$ isobaric chain using several SF parameterizations as indicated in the legend.

Routhian (3):

$$\hat{H}^\omega = \hat{H} - \boldsymbol{\omega}\hat{\mathbf{T}} + \frac{1}{2}\kappa\hat{\mathbf{T}}\hat{\mathbf{T}} \longrightarrow \hat{H} - [\boldsymbol{\omega} - \kappa\langle\hat{\mathbf{T}}\rangle]\hat{\mathbf{T}}. \qquad (4)$$

The Hartree approximation to (4) corresponds again to the isospin cranking model but with an effective, isospin dependent frequency. Within the HF approximation (no pairing) the model Routhian (4) gives rise to the following symmetry energy formula [2]:

$$E_{\rm sym} = \frac{1}{2}(\varepsilon + \kappa)T^2 + \frac{1}{2}\kappa T. \qquad (5)$$

Let us observe that, at variance with the standard textbook interpretation, part of the symmetry energy strength $a_{\rm sym} = \varepsilon + \kappa$ is related directly to the mean spacing of nuclear levels at the Fermi energy, ε, and not to the kinetic energy.

The schematic formula (5), can be tested within the fully self-consistent Skyrme-Hartree-Fock (SHF) model. This is due to the fact that the Skyrme force induced local energy density functional can be divided into isoscalar $t = 0$ and isovector $t = 1$ parts $E^{\rm Skyrme} = \sum_{t=0,1}\int {\rm d}^3\mathbf{r}\mathcal{H}_t(\mathbf{r})$ where

$$\mathcal{H}_t(\mathbf{r}) = C_t^\rho\rho_t^2 + C_t^{\Delta\rho}\rho_t\Delta\rho_t + C_t^\tau\rho_t\tau_t + C_t^J\mathbf{J}_t^2 + C_t^{\nabla J}\rho_t\boldsymbol{\nabla}\cdot\mathbf{J}_t. \qquad (6)$$

The coupling constants C are either density independent or depend only on the isoscalar density. The definitions of all local densities as well as the relations between the coupling constants C and the auxiliary parameters of the Skyrme force (SF) can be found, for example, in ref. [12].

In turn, the isoscalar $\mathbf{\Gamma}_0$ HF potential depends only on the isoscalar C_0 coupling constants, while the isovector HF potential $\mathbf{\Gamma}_1$ is defined ultimately by the isovector C_1 coupling constants. This property allows us to perform precise tests of eq. (5) using the following two-step procedure. In the first step we switch off the isovector potential $\mathbf{\Gamma}_1 \equiv 0$ by setting all $C_1 \equiv 0$. The calculated excitation energy with respect to the $N = Z$ (at $A = $ const) nucleus, $\Delta E_{\rm HF}^{(t=0)}$, can be compared to $\Delta E_{\rm HF}^{(t=0)} = \frac{1}{2}\varepsilon T^2$. In this way, one can extract the information about ε. In the next step, we perform full SHF calculations and compare the quantity $\Delta E_{\rm HF}^{(t=1)} \equiv \Delta E_{\rm HF} - \Delta E_{\rm HF}^{(t=0)}$ to $\Delta E_{\rm HF}^{(t=1)} = \frac{1}{2}\kappa T(T+1)$, giving us information about κ.

Our calculations, performed by using the code HFODD [13], show certain *generic* features which are illustrated in fig. 3. Namely, for a given value of A and for small values of $N - Z$, the values of $\varepsilon(A, T_z)$ change quite rapidly. They tend to stabilize for $N - Z \geq 8$ where $\varepsilon(A, T_z) \to \varepsilon(A)$. The value of the isoscalar-effective-mass scaled $\varepsilon^\star(A) = \frac{m^\star}{m}\varepsilon(A)$ is $55/A < \varepsilon^\star(A) < 66/A$, *i.e.* it lies within the experimental limits. The values of $\kappa(A, T_z)$ stabilize much faster, already for $N-Z > 4$ $\kappa(A, T_z) \to \kappa(A)$, showing that $\kappa(A)$ is free from the kinematic (shell) effects with surprisingly high accuracy. Let us observe that these features are common for all the tested parameterizations of the SF. The only exception is the SkO force with its unconventionally strong isovector component of the spin-orbit term. Such a spin-orbit term is inspired by the relativistic mean-field (RMF) models [14], and indeed our recent study shows that the RMF results follow qualitatively the SkO results [15].

All these features are nicely confirmed by large-scale calculations including isobaric chains of even-even nuclei from $A = 20$ till $A = 128$, see [16]. These calculations were performed specifically to establish the mass-number dependence of the nuclear symmetry energy, see fig. 4. The figure clearly shows that i) the values of $\varepsilon(A)$ heavily depend on the kinematics (shell effects); ii) in contrast, the values of $\kappa(A)$ are almost unaffected by the kinematics; iii) both $\varepsilon(A)$ and $\kappa(A)$ show clear surface $\sim 1/A^{4/3}$ dependence reducing the dominant volume term $\sim 1/A$. Considering only the two lowest-order expansion terms:

$$\varepsilon(A) = \frac{\varepsilon_V}{A} - \frac{\varepsilon_S}{A^{4/3}}; \qquad \kappa(A) = \frac{\kappa_V}{A} - \frac{\kappa_S}{A^{4/3}}, \qquad (7)$$

one can establish the volume and the surface contributions to ε and κ. For the case of the SLy4 force the ratio of the surface to the volume parameters equals $r_\varepsilon \equiv \varepsilon_S/\varepsilon_V \approx 1.56$ and $r_\kappa \equiv \kappa_S/\kappa_V \approx 1.45$, *i.e.* $r_\varepsilon \approx r_\kappa \approx 3/2$. The value of r_ε can be estimated based on the semi-classical formula of the level density developed for a diffuse-wall

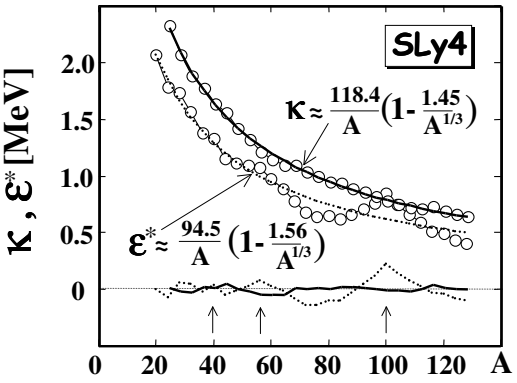

Fig. 4. The values of $\varepsilon^\star(A)$ and $\kappa(A)$ calculated for the isobaric chains of e-e nuclei from $A = 20$ till 128. The curves fitting the calculated points and their parameters are indicated in the figure. The differences between the smooth trends and the calculated points are also shown. These curves nicely illustrate that $\varepsilon^\star$ strongly depends on the shell effects while κ is essentially independent of the kinematics.

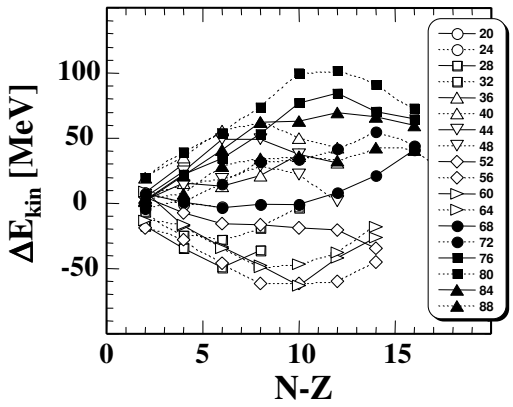

Fig. 5. Expectation value of the kinetic energy with respect to $N = Z$ nucleus, $\Delta E_{\rm kin} \equiv E_{\rm kin}(N-Z, A) - E_{\rm kin}(0, A)$, calculated using the SLy4-SHF approach for several isobaric chains of nuclei. Note that $\Delta E_{\rm kin}$ does not show any systematic trend when plotted *versus* $N - Z$.

potential well by Stocker and Farine [17]:

$$\varepsilon(A) \sim g(\epsilon_F)^{-1} \sim \frac{1}{A}\left(1 - \frac{\pi}{4k_{\rm F}^{(B)}}\frac{S_M}{V_M} + \dots\right), \quad (8)$$

where V_M and S_M denote the volume and the surface of nuclear matter distribution, respectively. The value of the bulk Fermi momentum is $k_{\rm F}^{(B)} \approx 1.36$ fm^{-1}.

The volume coefficient evaluated according to eq. (8) is unrealistic since it corresponds to the Fermi gas model estimate. However, assuming spherical geometry $\frac{S_M}{V_M} \approx \frac{3}{r_o A^{1/3}}$, one can expect a rather reliable estimate for the ratio r_ε. Adopting for $r_o \approx 1.14$ fm, *i.e.* the value consistent with the standard Skyrme force saturation density $\rho_0 \approx 0.16\,\mathrm{fm}^{-1}$, one obtains the ratio of the surface to volume contribution to the symmetry energy equal to $r_\varepsilon \approx \frac{3\pi}{4k_{\rm F}^{(B)} r_o} \approx 1.52$ which almost perfectly matches the value calculated for the SLy4 force.

All these facts seem to confirm very nicely the correctness of the symmetry energy formula (5), and in turn the reliability of the iso-cranking technique. Let us finally point out that the standard, Fermi gas model driven explanation of the symmetry energy coefficient as being partially due to the kinetic energy has no support in our calculations. Indeed, the expectation value of the SHF kinetic energy does not correlate with $N - Z$ as shown in fig. 5.

4 Summary

Applications of the isospin cranking model to *np*-pairing and the nuclear symmetry energy were briefly discussed. It is demonstrated that this generalized rotation gives an intuitive and simple understanding of the response of the isovector and isoscalar pair-fields with respect to the isospin degree of freedom.

It is also shown that the predictions of the iso-cranking model concerning the nuclear symmetry energy are consistent with the self-consistent SHF results. The arguments are given, that part of the symmetry energy strength, which is traditionally connected to the kinetic energy, is related in fact to the mean-level spacing. Moreover, it is demonstrated that the SHF isovector mean-potential can be characterized by an effective two-body interaction $v_{TT} = \frac{1}{2}\kappa\hat{\mathbf{T}}^2$ with unexpectedly high precision.

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